

# The general bi and tri-directional non-linear cross flow signal traffic in a digital network

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## Abstract

In this work we determine the sufficient and necessary conditions for having a bi and tri-directional non-linear cross flow in a digital network, acting on the matrix state of the optoelectronic or electronic gates set. We call matrix state to the configuration on the time to the states **A** and **B** of the gates so that the signals travel in the  $(i, j)$  directions without interference. For the signal model we consider a piece-wise non-linear functions which namely pulse train. In this paper we also make a theoretical extension to the same problem in three,  $n$ -dimensions and infinite dimensions.

Key word: Digital network, synchronisation.

## 1. Introduction

In 1997 E. Marchi solved the Manhattan problem of synchronisation applied to modern traffic science in two directions and in [68] completely, which is clearly one major advance the theory of traffic flow system. We have seen a great potential of future engineering applications in the mathematical LAUMAR system model. In this work we consider two digital signals which are flowing simultaneously in the  $(i, j)$  directions. The goal of this work is explorer the sufficient and necessary conditions for having a transparent grid acting on the matrix state of the electronic or optoelectronic gates set. We call matrix state to the configuration on the time of the states **A** and **B** of the gates so that the signals travel in the  $(i, j)$  directions without interference.

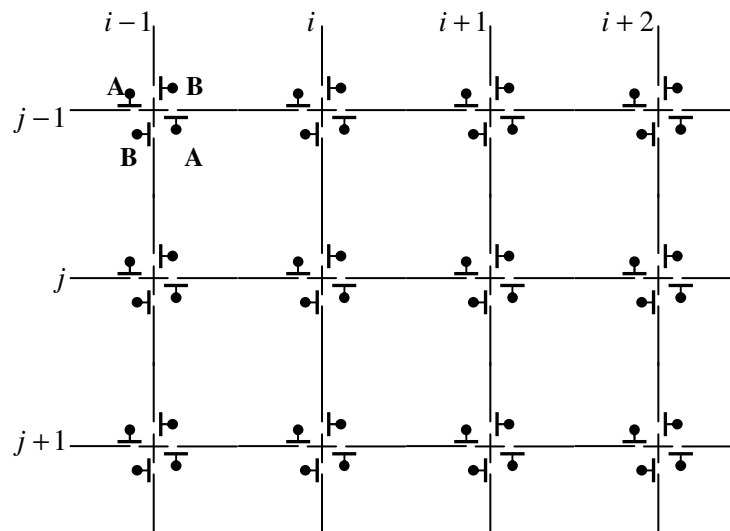


Fig. 1

We consider a  $m \times n$  rectangular interconnected plane grid network (Fig 1). We construct our network using a set of electronic or optoelectronic gates and transmission line.[66] These gates have been represented in a simple form by means of switches which have the following operation conditions.

When **A** contacts in any node  $(i, j)$  are closed, the digital signal travels by the  $i$  direction because the **B** contacts remain open during  $k$  time units and vice versa. Under this condition, the complete inversion of the electronic or optoelectronic gates is carried out at  $2k$  time units.

We don't say anything about the length between nodes but the inversion switch time it will concordance with the time to takes the signal in travel from a node to another. We know that the propagation signal speed in the grid will depend on the grid type and of its interconnection elements [67]. The inversion time operation of the electronic or optoelectronic gates will depend on the gate type used in each case.

## 2. Description of the grid: topology and operation.

Consider a  $m \times n$  rectangular interconnected plane grid network as the shown in the figure 1. All the transmission media, strip line, optical fiber, coaxial cable, etc. are one-way. In order to describe such a grid we introduce a  $(i, j)$  coordinate system. The way on the coordinates indexed by  $i:1, \dots, m$  has left-right direction and the way indexed by coordinate  $j:1, \dots, n$  has top-bottom direction. The intersection of the ways  $i$  and  $j$  is described by  $(i, j)$  node. The length of the conductor between intersections  $(i, j)$  and  $(i, j+1)$  is constant for each  $i$  and is  $l_{j+1} - f_{j+1}$  where  $f_{j+1}$  measures the distance in the  $i$  direction between two consecutive electronics gates around  $(i, j+1)$  node. Similarly, the length of the conductor between intersections  $(i, j)$  and  $(i+1, j)$  is constant for each  $j$  and is  $k_{i+1} - e_{i+1}$  where  $e_{i+1}$  measure the distance in the  $j$  direction between two consecutive electronics gates around  $(i+1, j)$  node. Consider for each intersection  $(i, j)$  a two-phase pass signal, which is a mechanism that allows the passage in one direction when the gate is open, whereas in the other direction the passage is forbidden. This is performed in a period of time, after which the situation is inverted for a period of time, which in general is different to the previous period. These periods are not necessarily periodically repeated.

We assume that the direction in which the signals are moving is the same for all signal on the transmission line indexed by  $i$  and we make the same assumption for the signals indexed with  $j$ . Then at the intersection  $(i, j)$  we indicate the two phases of the pass signal marking the direction by two periods as it is showed in the figure 2.

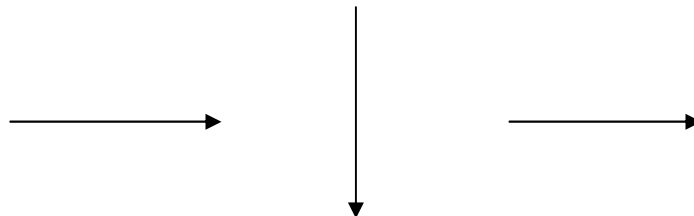


Fig. 2

The first indicates the permitted way in a period corresponding to the way with coordinate  $i$  where the left-right direction has the gate open. The second tells us analogously when the passing through is permitted on the way indexed by  $j$  (which is top-bottom direction).

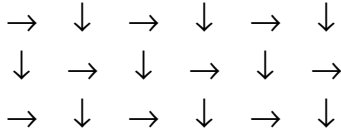
Now if a new index  $k$  tells us the period of changing state gate from left-right direction to top-bottom direction and vice versa, then we can consider a special  $(i, j)$  that for time  $t_{i\bar{j},2k}$  to time  $t_{i\bar{j},2k+1}$  at which the gate at the intersection  $i, j$  is open in the left-right direction and closed in the top-bottom direction and from the time  $t_{i\bar{j},2k+1}$  to  $t_{i\bar{j},2k+2}$  when the gate will be open for top-bottom direction and closed in the left-right direction.

Now if we would like to obtain general conditions for having a very good gate open wave in the grid we might ask the previous conditions for all the  $(i, j)$  such that  $i + j = \text{even}$  and for those  $i + j = \text{odd}$  we have that for time  $t_{ij,2k}$  to  $t_{ij,2k+1}$  is close in the left-right direction and open in the top-bottom direction and from the times  $t_{ij,2k+1}$  to  $t_{ij,2k+2}$  is open in the left-right direction.

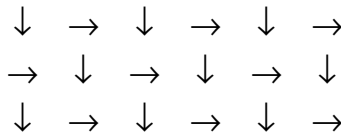
Now in order to have an order in the grid regarding the possibility of the open gate wave, we consider intuitively that for each  $k$ , the times are ordered in such a way that there exist times  $t_k$  such that

$$t_k < t_{ij,k} < t_{k+1}$$

Therefore if we write down the arrangement from  $2k$  to  $2k + 1$  we have for the grid



And from  $2k + 1$  to  $2k + 2$



Marchi E. has called these arrangements “LAUMAR systems”. At this point we would like to say that they might be related to vector fields moving accordingly in two phases.

The previous matter can be illustrated in a dynamics contest by means of the matrix state. By the case of the  $i + j = \text{even}$  then we have,

$$C_{ij(t)} = \begin{cases} 1 & t_{ij2k} \leq t < t_{ij2k+1} \\ 0 & t_{ij2k+1} \leq t < t_{ij2k+2} \end{cases}$$

and for those  $i + j = \text{odd}$

$$D_{ij(t)} = \begin{cases} 1 & t_{ij2k+1} \leq t < t_{ij2k+2} \\ 0 & t_{ij2k+2} \leq t < t_{ij2k+3} \end{cases}$$

in another way we indicate this in a more practical manner,

$$C_{ij(t)} = \left[ \begin{array}{ccc|c} 1 & 0 & 1 & \\ 0 & 1 & 0 & \\ 1 & 0 & 1 & \\ \hline & & & \end{array} \right] \quad t_{ij2k} \leq t < t_{ij2k+1}$$

and

$$D_{ij(t)} = \left[ \begin{array}{ccc|c} 0 & 1 & 0 & \\ 1 & 0 & 1 & \\ 0 & 1 & 0 & \\ \hline & & & \end{array} \right] \quad t_{ij2k+1} \leq t < t_{ij2k+2}$$

In general when the phase of the matrix control are not independent of the interconnection node then is going to be  $k(i, j)$ . This allows us to have dynamic state matrix which now has the property to describe completely the grid function. In our case that we impose the condition of the existence of special times  $t_k$  such that,

$$t_k \leq t_{ijk(i,j)} \leq t_{k+1}$$

for each  $k, i, j$ . This condition which is very general do not restrict the dynamics of the electronic system. In the case of two phases, we have a special particular case. In general we have the matrix,

$$C_{ij(t)} = \begin{cases} 1 & t_{ij2k(i,j)} \leq t < t_{ij2k(i,j)+1} \\ 0 & t_{ij2k(i,j)+1} \leq t < t_{ij2k(i,j)+2} \end{cases}$$

and

$$D_{ij(t)} = \begin{cases} 1 & t_{ij2k(i,j)+1} \leq t < t_{ij2k(i,j)+2} \\ 0 & t_{ij2k(i,j)+2} \leq t < t_{ij2k(i,j)+3} \end{cases}$$

At this point we would like to emphasise that is potentially important to introduce random matrices. This approach in our opinion is very interesting since it is possible to approach the percolation theory and different topics in modern physical problems.

### 3. Operation for finite pulse train.

Now if we consider a pulse train going on the  $[i, i+1]$  or  $[j, j+1]$  which are segments in a general way. The pulse train is described by the beginnings of the first pulse and the end of the last one.

The pulse train might move in a non-linear way, which is very general and the propagation speeds are not equal in every line segment. In a more realistic approach we have the case when the pulses form a train, say the pulse passing through  $(i, j)$ , at the interval of time measured by  $2k$  and  $2k + 1$  when  $i + j = \text{even}$ , possesses a pulse train which could change during time. Moreover we allow the inverse of the propagation velocity  $v$  to be a function of  $x$ . Therefore the pulse train at the intersection is measured by the first point to the right and the last point to the left of it which moves with linear movement given by

$$t = {}^1t_0^{ij} + {}^1v^{ij}(x_i)x_i$$

where,

$${}^1v^{ij}(x_i) = \frac{1}{{}^1V^{ij}(x_i)}$$

and  ${}^1V^{ij}(x_i)$  is the propagation velocity of the pulse train.

And

$$t = {}^2t_0^{ij} + {}^2v^{ij}(x_i)x_i$$

Where  ${}^1t_0^{ij}$  is the time when the initial position in the block  $(i, j)$  of the right point leading the wave train starts. In other words, for arbitrary  $\bar{x}_i$  the point the associated time is,

$${}^*\bar{t}_i = {}^1t_0^{ij} + {}^1v^{ij}(\bar{x}_i)\bar{x}_i$$

which for the first point, the time is,  ${}^*\bar{t}_i = {}^1t_0^{ij} + {}^1v^{ij}({}^1\bar{x}_i){}^1\bar{x}_i$ . At this time the second point to the left has the position  ${}^2\bar{x}_i$  by solving the equation in the variable  ${}^2\bar{x}_i$ ,

$${}^*\bar{t}_i = {}^1t_0^{ij} + {}^1v^{ij}(\bar{x}_i)\bar{x}_i = {}^2t_0^{ij} + {}^2v^{ij}({}^2\bar{x}_i){}^2\bar{x}_i$$

We assume that the segment  $[\bar{x}_i^1, \bar{x}_i^2]$  given at the same time which fulfils the above equation has the length less than  $\min l_i$ , that is to say the lengths of the wave train are less than the lengths of the blocks ways, during the gates are open.

Take for example that,

$$\begin{aligned}
{}^2v^{ij} &= c \quad c > 0 \\
{}^*t_i &= {}^2t_0^{ij} + c \left( {}^2x_i \right) \quad \text{or} \\
{}^2x_i &= \frac{{}^*t_i - {}^2t_0^{ij}}{c}
\end{aligned}$$

Another example, more difficult is if,

$$\begin{aligned}
{}^2v^{ij} &= cx_i \quad c > 0 \\
{}^*t_i &= {}^2t_0^{ij} + c \left( {}^2x_i \right)^2 \quad \text{or} \\
{}^2x_i &= \sqrt{\frac{{}^*t_i - {}^2t_0^{ij}}{c}}
\end{aligned}$$

If we assume that,

$$C_i^{\bar{i}} \left[ \sum_{p=1}^{\bar{i}-1} l_p + x_i \right] + \bar{C}_i^{\bar{i}} = {}^1v_i^{ij}(x_i)$$

which is linear piece-wise function in each block way, such that

$$C_i^{\bar{i}} \left[ \sum_{p=1}^{\bar{i}} l_p \right] + \bar{C}_i^{\bar{i}} = C_i^{\bar{i}+1} \left[ \sum_{p=1}^{\bar{i}} l_p \right]$$

or

$$\left( C_i^{\bar{i}} - C_i^{\bar{i}+1} \right) \left[ \sum_{p=1}^{\bar{i}} l_p \right] = \bar{C}_i^{\bar{i}+1} - \bar{C}_i^{\bar{i}}$$

for  $0 \leq y \leq k_{\bar{i}}$  and for  $0 \leq x \leq k_{\bar{i}}$ , that is to say, the movement is non-linear and continuous arbitrary function

$$\sum_{p=1}^{\bar{i}} v_p^{ij} + {}^1t_0^{ij} = {}^2t_0^{ij} + \sum_{p=1}^{\bar{i}} v_p^{ij}(k_p) + {}^2v_i^{ij} x$$

with different speeds for different blocks, then for  $\bar{x}_i^1 = \sum_{p=1}^{\bar{i}} k_p$

Since we assumed that the length of pulse train  $[{}^2x, {}^1x]$  at any time is less than the length of any block. Therefore at the intersection  $(i, j)$  with  $i + j = \text{even}$  the period marked with  $2\bar{k}$  and  $2\bar{k} + 1$  the inequalities expressed for having the entire wave or pulse train passing during the open period of the gate in the direction left-right is now given by

$$t_{ij,2\bar{k}} \leq {}^1t_0^{ij} + \sum_{p=1}^{i-1} {}^1v_0^{ij}(k_p) - {}^1v_i^{ij}(e_i)$$

$${}^2t_0^{ij} + \sum_{p=1}^{i-1} {}^2v_p^{ij}(k_p) \leq t_{ij,2\bar{k}+1}$$

Following in this way in all the next gate such a wave train passing during the open period it has to satisfy

$$t_{i+n-1,j,2\bar{k}+n} \leq {}^1t_0^{ij} + \sum_{p=1}^{i+n-1} {}^1v_0^{ij}(k_p) - {}^1v_{i+n-1}^{ij}(e_{i+n-1})$$

and

$${}^2t_0^{ij} + \sum_{p=1}^{i-1} {}^2v_p^{ij}(k_p) \leq t_{ij,2\bar{k}+1}$$

Similarly for the pulse train going in the other direction top-bottom, with the variable  $y$ . The wave train of a package is represented by the points  $[{}^1y_i, {}^2y_j]$  which we assume that are less in length than the minimum block in that direction. Again we might consider that the inverse of the velocity might change in each block, therefore the movement of both points enclosing the wave train change in a non-linear form by the equations

$$t = {}^1t_0^{ij} + {}^1v^{-ij}(y_j)y_j$$

and

$$t = {}^2t_0^{ij} + {}^2v^{-ij}(y_j)y_j$$

where

$${}^1v^{-ij}(y_j) = \frac{1}{{}^1\bar{V}^{ij}(y_j)}$$

and  ${}^1V^{ij}(y_j)$  is the propagation speed of the pulse train. The same for second equation for the last point of the pulse train. Therefore,

$$t = {}^2t_0^{ij} + {}^2v^{-ij}(y_j)y_j$$

where again

$${}^2v^{-ij}(y_j) = \frac{1}{{}^2\bar{V}^{ij}(y_j)}$$

is the inverse of the propagation velocity of the pulse train of the backward part of the wave.

For the point  $\bar{y}_j$  the associated time when the leading point of the pulse train is given by,

$${}^* \bar{t}_j = {}^1 \bar{t}_0 + {}^1 \bar{v}^{ij} (\bar{y}_j) \bar{y}_j$$

At this time for the backward or in the top has the position  ${}^2 \bar{y}_j$  which is a root in the variable  ${}^2 \bar{y}_j$  of

$${}^* \bar{t}_j = {}^1 \bar{t}_0 + {}^1 \bar{v}^{ij} (\bar{y}_j) {}^1 \bar{y}_j = {}^2 \bar{t}_0 + {}^2 \bar{v}^{ij} (\bar{y}_j) {}^2 \bar{y}_j.$$

We assume that the pulse train determined by step functions given by  $[{}^1 \bar{y}_j, {}^2 \bar{y}_j]$  obtained by the previous equation, has the length less than  $\min k_j$ . In other words the pulse train has less length that the length of the block ways, meanwhile the gates are opens.

An illustrative example is given as follows.

$$\begin{aligned} {}^2 \bar{v}^{ij} &= d \quad d > 0 \\ {}^* \bar{t}_j &= {}^2 \bar{t}_0 + c \left( {}^2 \bar{y}_j \right) \quad \text{or} \\ {}^2 \bar{y}_j &= \frac{{}^* \bar{t}_j - {}^2 \bar{t}_0}{d} \end{aligned}$$

A more suitable example, which has some more difficulties is, if

$$\begin{aligned} {}^2 \bar{v}^{ij} &= d y_j \quad d > 0 \\ {}^* \bar{t}_j &= {}^2 \bar{t}_0 + d \left( {}^2 \bar{y}_j \right)^2 \quad \text{or} \\ {}^2 \bar{y}_j &= \sqrt{\frac{{}^* \bar{t}_j - {}^2 \bar{t}_0}{d}} \end{aligned}$$

taking that,

$$D_j^{\bar{j}} \left[ \sum_{p=1}^{\bar{j}-1} l_p + y_j \right] + \bar{D}_j^{\bar{j}} = {}^1 \bar{v}_j^{ij} (y_j)$$

we have that this function is a linear piece-wise function in each block way, such that,

$$D_j^{\bar{j}} \left[ \sum_{p=1}^{\bar{j}} l_p \right] + \bar{D}_j^{\bar{j}} = \bar{D}_j^{\bar{j}+1} \left[ \sum_{p=1}^{\bar{j}} l_p \right]$$

or

$$(D_j^{\bar{j}} - D_j^{\bar{j}+1}) \left[ \sum_{p=1}^{\bar{j}} l_p \right] = D_j^{\bar{j}+1} - D_j^{\bar{j}}$$

The fact given by this equality says something interesting from an electrical point view. Many other examples can be presented.

A similar analysis to that performed for the synchronisation of the pulse train with the open gates wave in the other direction can be obtained for the y's. Obtaining finally for the pulse train passing at  $(i, j)$  with  $(i + j) = \text{odd}$  at the time with index  $k$  in the following period of time

$$t_{ij+u, 2k+1+u} \leq t_0^{ij} + \sum_{p=1}^{j+u-1} v_p^{-ij} (l_p) - v_{j+u}^{-ij} (l_{j+u} - f_{j+u})$$

where it is clear that each inverse velocity is defined in its proper interval, which is given from the initial position to the end.

And

$$\sum_{p=1}^{j+u} v_p^{-ij} (l_p) + y_0^{ij} \leq t_{ij+u, 2k+2u}.$$

In such a way we have obtained sufficient conditions, which are also necessary for having the existence open gates wave for the complete LAUMAR schemes of the marked car by  $(i, j)$  and together with the pulse train.

#### 4. Co-ordination for the simultaneously bi-directional pulse train

Before to get the general condition for co-ordination of the simultaneously bi-directional pulse train, we consider the intersection  $(i, j-1)$  for the previous given  $i$  and  $j$ , a pulse going bottom then since  $i-1+j = \text{odd}$  for the same conditions as the previous pulse going left-right direction we have for the linear movement given by,

$$t = y_0^{ij-1} + v^{-ij-1} y_j$$

it has to fulfil

$$t_{ij-1, 2k} \leq y_0^{ij-1} + v^{-ij-1} \left[ \sum_{s=1}^{j-1} l_s - f_{j-1} \right]$$

and

$$y_0^{ij-1} + v^{-ij-1} \left[ \sum_{s=1}^{j-1} l_s \right] \leq t_{ij-1, 2k+1}$$

For the next change in the state of the gate which is  $(i, j)$  we should have

$$t_{ij-1,2k+1} \leq y_0^{ij-1} + \bar{v}^{ij} \left[ \sum_{s=1}^j l_s - f_j \right]$$

and

$$y_0^{ij} + \bar{v}^{ij} \left[ \sum_{s=1}^j l_s \right] \leq t_{ij-1,2k+2}.$$

Now it is to see the whole synchronisation for many pulses, (pulse train) going in a straight way. Consider for each  $(i, j)$  in the grid at the interval given by  $2k$  and  $2k+1$  a pulse going in the left-right direction if  $i+j = \text{even}$  and top-bottom if  $i+j = \text{odd}$ . The first ones have to satisfy

$$t_{ij,2\bar{k}} \leq x_0^{ij} + v^{ij} \left[ \sum_{s=1}^i k_s - e_j \right]$$

and

$$x_0^{ij} + v^{ij} \left[ \sum_{s=1}^j k_s \right] \leq t_{ij,2\bar{k}+1}.$$

Such a pulse in order to get all the open gates in all the next state gates has to fulfil the equations

$$t_{i+nj,2\bar{k}+n} \leq t_0^{ij} + v^{ij} \left[ \sum_{s=1}^{i+n} k_s - e_{j+n} \right]$$

and

$$x_0^{ij} + v^{ij} \left[ \sum_{s=1}^{j+n} k_s \right] \leq t_{i+nj,2\bar{k}+1+n}$$

for all  $n$  such that  $i+n \geq 1$

$$t_{ij,2\bar{k}} \leq y_0^{ij} + \bar{v}^{ij} \left[ \sum_{s=1}^j l_s - f_j \right].$$

Similarly the pulses in the "starting" positions  $(i, j)$  such that  $i+j = \text{odd}$ , therefore we have

$$y_0^{ij} + \bar{v}^{ij} \left[ \sum_{s=1}^j l_s \right] \leq t_{ij-1,2\bar{k}+1}.$$

And moreover for all the remaining positions we have to have

$$t_{ij+u,2\bar{k}+u} \leq y_0^{ij} + \bar{v}^{ij} \left[ \sum_{s=1}^{i+u} l_s - f_{j+n} \right]$$

and

$$y_0^j + v^{-ij} \left[ \sum_{s=1}^{j+u} l_s \right] \leq t_{ij+u, 2\bar{k}+u}$$

for all  $u$  such that  $j+u \geq 1$ .

Given a pulse in the position  $(i, j)$  with  $i+j = \text{even}$  at the time between  $2k$  and  $2k+1$ , passing through the intersection in open gate state, then such a pulse between the times expressed by  $2\bar{k}$  and  $2\bar{k}-1$  it was around or crossing the intersection  $(i, j-2(k-\bar{k}))$  if  $i-2(k-\bar{k}) \geq 1$  and in a virtual position given by the same number if such amount is negative. Similarly if  $i+j = \text{odd}$  the position between the periods considered by  $2\bar{k}$  and  $2\bar{k}-1$  it was  $(i, j-2(k-\bar{k}))$ .

Now we have taken in our model a simple case going left-right and top-bottom, but in the most general case the difficulties for the pulses going in the left-right direction are now becoming for the intersection  $(i, j)$  with  $i+j = \text{even}$  at the period  $2k, 2k+1$ .

$$t_{ij, 2k} \leq t_0^{ij} + v^{ij} \left[ \sum_{p=1}^i k_p \right]$$

and

$$t_0^{ij} + v^{ij} \left[ \sum_{p=1}^i k_p - f_j \right] \leq t_{ij, 2k+1}$$

since now we have the state gates in inverted position, the firsts are in the position  $\left[ \sum_{p=1}^i k_p, \sum_{p=1}^j l_p \right]$ , the second are at  $\left[ \sum_{p=1}^i k_p - e_i, \sum_{p=1}^j l_p \right]$ . Similarly for a pulse going top-bottom direction.

Until now we have considered that the pulses were represented by points which moves according a linear function.

Now we are going to generalise it by considering a pulse train or wave pocket for the movement of a package of pulses going all together. Moreover the pulse train might move in a piece-wise linear way and the speeds are not equals in all the blocks.

The previous study can be extended when the pulses do not move along represented by points. In a more real approach we have the case when the pulses form a pocket of pulses or pulse train, say the pulse passing through  $(i, j)$  at the interval of time measured by  $2k, 2k+1$  with  $i+j = \text{even}$  possesses a pulse train which could change during time. Moreover we allow the inverse of the velocity  $v$  to be function of  $x$ . Therefore the pocket of pulses at that intersection is measured by the first to the right and last to the left of it which moves with linear movement giving by

$$t = {}^1 t_0^{ij} + {}^1 v^{ij}(x_i) x_i$$

and

$$t = {}^2 t_0^{ij} + {}^2 v^{ij}(x_i) x_i$$

in other word at point  $x$  for the first point the time it is

$${}^1t_0^{ij} + {}^1v^{ij}(\bar{x}_i)\bar{x}_i.$$

At this time the second point to the left has the position  $x$  expressed by solving the equation

$${}^1t_0^{ij} + {}^1v^{ij}(\bar{x}_i)\bar{x}_i = {}^2t_0^{ij} + {}^2v^{ij}({}^2\bar{x}_i){}^2\bar{x}_i$$

We assume that the segment  $[{}^1\bar{x}_i, {}^2\bar{x}_i]$  has the length less than  $\min k_i$  that is to say the lengths of the pulse train are less than the length of the blocks.

If we assume that

$$\left[ \sum_{p=1}^{i-1} k_p + x \right] {}^1v^{ij} = {}^1v_i^{ij}$$

for  $0 \leq x \leq k_i$ , and

$$\left[ \sum_{p=1}^{i-1} k_p + x \right] {}^2v^{ij} = {}^2v_i^{ij}$$

for  $0 \leq x \leq k_i$ , that is to say, the movement is piece-wise linear with different speeds for different blocks, then for  ${}^1\bar{x}_i = \sum_{p=1}^i k_p$ ,

$$\sum_{p=1}^i v_p^{ij} + {}^1t_0^{ij} = {}^2t_0^{ij} + \sum_{p=1}^i v_p^{ij} k_p + {}^2v_i^{ij} {}^2\bar{x}_i$$

since we assumed that the length of pulse train  $[{}^1x_i, {}^2x_i]$  at any time is less than length of any block.

Therefore at the intersection  $(i, j)$  which  $i + j = \text{even}$  a the period marked with  $2\bar{k}$  and  $2\bar{k} + 1$  the inequalities expressed for having the entire pulse train passing during the open gate period in the left-right direction, is now given by

$$t_{ij, 2\bar{k}} \leq {}^1t_0^{ij} + \left[ \sum_{p=1}^{i-1} (v_0^{ij} k_p) - {}^1v_i^{ij} e_i \right]$$

and

$${}^2t_0^{ij} + \left[ \sum_{p=1}^i ({}^2v_p^{ij} k_p) \right] \leq t_{ij, 2\bar{k}+1}$$

Following in this way, in all the next states of the gate such a pulse train passing during the open gate period it has to satisfy

$$t_{i+n-1j,2\bar{k}+n} \leq {}^1t_0^{ij} + \left[ \sum_{p=1}^{i+n-1} ({}^1v_0^{ij} k_p) - {}^1v_{i+n-1}^{ij} e_{i+n-1} \right]$$

and

$${}^2t_0^{ij} + \left[ \sum_{p=1}^{i+n} ({}^2v_p^{ij} k_p) \right] \leq t_{i+n-1j,2\bar{k}+1+n} .$$

Similarly for the pulses going in the other top-bottom direction, with the variable  $y$ . The pulse train is represented by the points  $[{}^1y_j, {}^2y_j]$  which we assume that all are less in length than the minimum blocks in that direction. Again we might consider that the inverse of the velocity might change in each block, therefore the movement of the both points enclosing the pulse train change in a piece-wise linear form by the equation

$$t = {}^1t_{0y}^{ij} + {}^1v^{-ij}(y_j)y_j$$

and

$$t = {}^2t_{0y}^{ij} + {}^2v^{-ij}(y_j)y_j .$$

A similar analysis as that performed for the synchronisation of the pulse train with the open gate wave in the other direction can be obtained for the  $y$ 's. Obtaining, finally for the pulse train passing at  $(i, j)$  with  $i + j = \text{odd}$  at the time with index  $k$  in the following period of time

$$t_{ij+u,2k+1+u} \leq {}^1t_{0y}^{ij} + \left[ \sum_{p=1}^{j+u-1} ({}^1v_p^{-ij} l_p) - {}^1v_{j+u}^{-ij} f_{j+u} \right]$$

and

$${}^2t_0^{ij} + \left[ \sum_{p=1}^{i+n} ({}^2v_p^{ij} k_p) \right] \leq t_{i+n-1j,2\bar{k}+1+n} .$$

## 5. Co-ordination of the pulse train in three dimensions

Consider now a grid of the three dimension where each vertex is given by  $(i, j, k)$  where  $i:1,2,\dots,m$ ,  $j:1,2,\dots,n$  and  $k:1,2,\dots,p$ . This point is the intersection of the transmission lines which are explained by taking two fixed coordinates and changing the other. In this way we have three transmission lines passing at each point.

All of them are presented as

$$(i, j, k) \quad (i+r, j, k) \quad r=1,2,\dots$$

in the  $i$  direction, in the  $j$  direction we have

$$(i, j, k) \quad (i, j+r, k) \quad r=1,2,\dots$$

and finally the movement in the other direction is given by

$$(i, j, k) \quad (i, j, k + r) \quad r = 1, 2, \dots$$

Here following the material of previous paragraphs, now the three equations of movements in the three perpendicular directions are given by

$$\begin{aligned} t &= {}^1t_0^{ijk} + {}^1v^{ijk}(x_i)x_i \\ t &= {}^2t_0^{ijk} + {}^2v^{ijk}(y_j)y_j \\ t &= {}^3t_0^{ijk} + {}^3v^{ijk}(z_k)z_k \end{aligned}$$

were  ${}^1t_0^{ijk}$ ,  ${}^2t_0^{ijk}$  and  ${}^3t_0^{ijk}$  are the initial times. The  $v'$  are the inverse velocities in each direction. Then now we introduce the times when the state of the gate switches. Now instead of having two phases, we have three phases, are for each directions. Then let  $t_{ijkl}$  to be the time that the state gate switches at the point  $(i, j, k)$ . When  $l \equiv 1 \pmod{3}$  the state gate is open the direction  $i$ . We remind the reader for the definition of  $\pmod{3}$  is

$$a \equiv b \pmod{3} \text{ if } a - b = 3r \text{ for integer } r$$

In the second instance when the state gate switches in the direction of the second axis  $j$  is turning at  $t_{ijkl} \quad l \equiv 2 \pmod{3}$  and finally in the remaining direction is  $t_{ijkl} \quad l \equiv 3 \pmod{3}$ .

Therefore in the corresponding direction we have open gate as shown in the following scheme,

$$\begin{aligned} t_{ijk3l} &\Rightarrow t_{ijk3l+1} \text{ open gate in the } i \text{ direction} \\ t_{ijk3l+1} &\Rightarrow t_{ijk3l+2} \text{ open gate in the } j \text{ direction} \\ t_{ijk3l+2} &\Rightarrow t_{ijk3l+3} \text{ open gate in the } k \text{ direction} \end{aligned}$$

we define open gate state when the pulse train can happen through the gates. These LAUMAR systems can be co-ordinated in a matrix way, namely: this can be illustrated in a dynamics contest by means of the tensor state. By the case of  $i + j + k = 1 \pmod{3}$

$$\begin{aligned} C_{ijk}(t) &= \begin{cases} 1 & t_{ijk3l} \leq t < t_{ijk3l+1} \pmod{3} \\ 0 & t_{ijk3l+1} \leq t < t_{ijk3l+2} \pmod{3} \end{cases} \\ D_{ijk}(t) &= \begin{cases} 1 & t_{ijk3l+1} \leq t < t_{ijk3l+2} \pmod{3} \\ 0 & t_{ijk3l+2} \leq t < t_{ijk3l} \pmod{3} \end{cases} \end{aligned}$$

$$E_{ijk}(t) = \begin{cases} 1 & t_{ijk3l+2} \leq t < t_{ijk3l} \pmod{3} \\ 0 & t_{ijk3l} \leq t < t_{ijk3l+1} \pmod{3} \end{cases}$$

In other way and in a more schematic way we can introduce and indicate these functions by tensors of high dimension. They are

$$C_{ijk}(t) = \begin{matrix} k=1 \\ \left[ \begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & . \\ 0 & 1 & 0 & 0 & 1 & . & . \\ 1 & 0 & 0 & 1 & . & . & . \\ 0 & 0 & 1 & . & . & . & . \\ 0 & 1 & . & . & . & . & . \\ 1 & . & . & . & . & . & . \end{array} \right] \end{matrix} \quad t_{ijk3l} \leq t < t_{ijk3k+1}$$

$$C_{ijk}(t) = \begin{matrix} k=2 \\ \left[ \begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & . \\ 0 & 1 & 0 & 0 & 1 & . & . \\ 1 & 0 & 0 & 1 & . & . & . \\ 0 & 0 & 1 & . & . & . & . \\ 0 & 1 & . & . & . & . & . \\ 1 & . & . & . & . & . & . \end{array} \right] \end{matrix} \quad t_{ijk3l+1} \leq t \leq t_{ijk3l+2}$$

$$C_{ijk}(t) = \begin{matrix} k=3 \\ \left[ \begin{array}{cccccc} 1 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & . \\ 0 & 1 & 0 & 0 & 1 & . & . \\ 1 & 0 & 0 & 1 & . & . & . \\ 0 & 0 & 1 & . & . & . & . \\ 0 & 1 & . & . & . & . & . \\ 1 & . & . & . & . & . & . \end{array} \right] \end{matrix} \quad t_{ijk3l+2} \leq t \leq t_{ijk3l+3}$$

In a similar form it is possible to repeat the material of one and two dimension introducing a dimension for tensors. It is easy to extend all results as printed in the previous paragraph.

The other  $D$  and  $E$  matrices can be written accordingly. The open gates coordination in the direction  $i$  is given by

$$t_{ijk,3l} = t_0^{ijk} + \sum_{p=1}^{i-1} {}^1v_p^{ijk}(k_p) - {}^1v_i^{ijk}(e_i)$$

$${}^2t_0^{ijk} + \sum_{p=1}^{i-1} {}^2v_p^{ijk}(k_p) \leq t_{ijk,3l+1} .$$

Here they are the points to the left and right in the pulse train.

## 6. Co-ordination of theoretical pulse train in n and infinite directions

In the case of  $r$  dimensions we have that the movement equations in the  $s$ -direction is given by

$$t = {}^s t_0^{m_1, m_2, \dots, m_r} + {}^s v^{m_1, m_2, \dots, m_r} (x_{m_s}) x_{m_s} \quad 1 \leq s < r$$

and where the open gate under way in the  $s$  direction in the interval,

$$t = t_{sm_1, \dots, sm_{s+1}, \dots, m_r} \rightarrow t_{m_1, \dots, sm_{s+s}, \dots, m_r}$$

the gate is open in the direction  $s \bmod r$ . In the numerable infinite case,

$${}^s t_0^{m_1, \dots, m_s, \dots} + {}^s v_0^{m_1, \dots, m_s, \dots} (x_{m_s}) x_{m_s} \quad 1 \leq s < \infty$$

and the open state gate is under way during the times

$$t_{sm_1, \dots, sm_{s+1}, \dots} \rightarrow t_{m_1, \dots, sm_{s+s}, \dots}$$

such that

$$\sum_{s=1}^{\infty} (t_{sm_1, \dots, sm_{s+s}, \dots} - t_{sm_1, \dots, sm_{s+1}, \dots}) < \infty$$

and also, which says that the state gate changes are possible.

$$\sum_{m_1 \dots m_r \dots} \bar{t}_{m_1 \dots m_r \dots} < \infty$$

## 7. Final remark

We would like to remark that our approach of the synchronisation of the pulse train in different directions either one or two direction is original. Moreover it is possible to extend it to triangular arrays or to rectangular are with diagonals as for example, we have done it for the case of  $7 \times 7$  with diagonal using backtracking technique. There in a program due to Adversa and Marchi around 12000 different LAUMAR systems. All this material see Marchi [68], beside to the application to electronic digital network, it might be of interest to related topics as for example in percolation and computing in non-linear media and automata collectives. See A. Adamatzky.

It is interesting to see the possibility to develop in Fourier series the platoon which is given by the step function with extremes  $^1t$  and  $^2t$ . About it and many other matters we will present in the future different presentations.

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